سرمينر

Information Theory and Coding (CM303)

Midterm Exam (40%)
07 June, 2015

Instructor: MEng. Hosam Almqadim | **Time Allowed: 2 hours**

Q1. What is the maximum entropy H(s) for Binary Discrete Memoryless Source (BDMS)? Prove your answer? **(10 points)**

Q2. A telegraph source having two symbols, dot and dash. The dash duration is 0.6 seconds; and the dot duration is two third of the dash duration. The probability of the dot occurring is twice that of the dash, and the time between symbols is 0.2 seconds. Calculate the information rate of the telegraph source? **(10 points)**

Q3. A discrete memoryless source has an alphabet of seven symbols with probabilities for its output as described in following table:

Symbol	S_0	S_1	S_2	S_3	S_4	S_5	S_6
Probability	0.25	0.25	0.125	0.125	0.125	0.0625	0.0625

1. Construct a Shannon-Fano code for the source and calculate the efficiency of coding? **(10 points)**
2. Construct a Huffman code for the source and calculate the efficiency of coding? And compare the results? **(10 points)**

Good luck!

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Answer of Midterm Exam (40%)

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Q1. Answer:

Since it BDMS, then the source has two symbols s1 and s2. Let the probability of s1 is p(s1)=a then the probability of s2 is p(s2)=1-a. The entropy H(s) of this source:

$H(s) = -a\log_2(a) - (1-a)\log_2(1-a)$ $-a\log_2 a$

Note that when $a=0 \rightarrow H(s) = 0$

$a=1 \rightarrow H(s) = 0$

The maximum entropy can be found by the differentiation of H(s):

$$\frac{dH(s)}{da} = \frac{d(-a\log_2(a) - (1-a)\log_2(1-a))}{da} = -\left[a\cdot\frac{1}{a} - \log_2 a\right] - \left[1-a\frac{-1}{(1-a)} - \log_2(1-a)\right]$$

$$= -1 - \log_2(a) + \log(1-a) = \log\left(\frac{1-a}{a}\right)$$

$$\frac{dH(s)}{da} = -\log_2(a) + \log_2(1-a)$$

$$\frac{dH(s)}{da} = \log_2\left(\frac{1-a}{a}\right)$$

The maximum is found when $\frac{dH(s)}{da} = 0$

$$\log_2\left(\frac{1-a}{a}\right) = 0 \text{ when } \frac{1-a}{a} = 1$$

$\therefore$ a=0.5

Which means when a=0.5 H(s) is maximum

$H(s) = -0.5\log_2(0.5) - (1-0.5)\log_2(1-0.5) = 1$ bit/symbol

Good luck!

Q2. Answer:

Given that:
1. Dash duration: 0.6 sec.
2. Dot duration: $2/3 \times 0.6 = 0.4$ sec.
3. P (dot) = 2 P (dash).
4. Space between symbols is 0.2 sec.

Information rate =? $I_s = R_s * H(s)$

1. Probabilities of dots and dashes:

Let the probability of a dash be "P". Therefore the probability of a dot will be "2P". The total probability of transmitting dots and dashes is equal to 1.

$\therefore$ P (dot) + P (dash) = 1

$\therefore$ P + 2P = 1 $\therefore$ P = 1/3

$\therefore$ Probability of dash = 1/3

And Probability of dot = 2/3

2. Average information H (X) per symbol:

$\therefore$ $H(X) = P(\text{dot}) \cdot \log_2 [1/P(\text{dot})] + P(\text{dash}) \cdot \log_2 [1/P(\text{dash})]$

$\therefore$ $H(X) = (2/3) \log_2 [3/2] + (1/3) \log_2 [3] = 0.3899 + 0.5283 = 0.9182$ bits/symbol.

3. Symbol rate (Number of symbols/sec.):

The total average time per symbol can be calculated as follows:

Average symbol time $T_s = [T_{DOT} \times P(DOT)] + [T_{DASH} \times P(DASH)] + T_{space}$

$\therefore$ $T_s = [0.4 \times 2/3] + [0.6 \times 1/3] + 0.2 = 0.6667$ sec/symbol.

Hence the average rate of symbol transmission is given by:

$R_s = 1/T_s = 1.5000$ symbols/sec.

4. Information rate (R_I):

$R_I = R_s \times H(s) = 1.5000 \times 0.9182 = 1.72$ bits/sec.

1.3773

$\frac{\ln 3/2}{\ln 2}$

Good luck!

Q3. Answer:

1. Shannon-Fano code:

Symbols	Probability	Step 1	Step 2	Step 3	Step 4	Code word
S_0	0.25	0	0			00
S_1	0.25	0	1			01
S_2	0.125	1	0	0		100
S_3	0.125	1	0	1		101
S_4	0.125	1	1	0		110
S_5	0.0625	1	1	1	0	1110
S_6	0.0625	1	1	1	1	1111

Average code word length (L): $\mathbf{L} = \sum_{k=0}^{6} \text{pi} \times \text{ni}$

$= (0.25 \times 2) + (0.25 \times 2) + (0.125 \times 3) + (0.125 \times 3) + (0.125 \times 3) + (0.0625 \times 4) + (0.0625 \times 4)$

$= 2.6250$ bits/message

Entropy of the source (H):

$$H(s_i) = \sum_{k=0}^{6} p_i \times \log_2(1/p_i)$$

$=0.25 \log_2 (1 / 0.25) + 0.25 \log_2 (1 / 0.25) + 0.125 \log_2 (1 / 0.125) + 0.125 \log_2 (1 / 0.125) + 0.125 \log_2 (1 / 0.125) + 0.0625 \log_2 (1 / 0.0625) + 0.0625 \log_2 (1 / 0.0625) = 2.6250$ bits/symbols

Code efficiency $\eta = \frac{H}{L} \times 100 = \frac{2.625}{2.625} \times 100$

$\therefore \quad \eta = 100\%$

Good luck!